Design of a flexural hinge mechanism for Turbo Slits

Michael Hillman

Diamond Light Source Ltd

Harwell Science & Innovation Campus

Didcot, OX11 0DE, UK

michael.hillman@diamond.ac.uk

Abstract - The I20 beamline at Diamond Light Source (DLS) requires a set of vertical "turbo" slits which can be scanned across a horizontal x-ray fan. To achieve a fast scan rate, requires a precision slit mechanism which is both light weight and compact. The design solution is to use a 4 bar flexural hinge geometry. Several different approaches may be used to design / analyse flexural hinges, including equations based on a simplified model of the hinge, FEA analysis either of the hinge or a full FEA analysis of the overall design, empirical values based on previous designs and mechanism analysis. This case study then presents the approach used to design the hinge mechanism for the turbo slits, using simple analytic equations to obtain the moments and stresses at the hinge, "large" angle kinematic mechanism analysis to optimise the overall movements of the mechanism, correlation of hinge parameters to give the optimum design and validation of forces, stresses & strains using a full FEA model. The slit mechanism has been built and the deflection / load performance experimentally validated. The scanning mechanism incorporating the slits has been demonstrated in the laboratory.

Keywords: FEA, Flexure, Flexural Hinge

1. Introduction

Flexures, also known as flexural hinges have been described from as long ago as 1937 (Eastman 1937). They are used to provide precise movements with minimum friction and backlash, and with a small range of motion. Widely used on scientific instrumentation they are now also used in consumer equipment such as hard disk drives.

We refer here primarily to monolithic flexures, where the hinge is machined from a solid blank. Flexural hinges may also be constructed from a clamped assembly, including flat spring strips, allowing a larger range of motion. Similar to flexural hinges, but used more for benefits of mass production and lower cost are plastic "living hinges" (Web-1), best known for their use in plastic bottle top closures.

The design of flexure mechanisms can be approached in a number of ways. With the easy availability of finite element analysis (FEA) software, such as Ansys™, it is tempting to construct a solid model "in silico" and analyse in 3D. We believe that there is benefit in carrying out the initial design using simple analytic equations, before validating the final design with FEA.

The design approach is illustrated by a case study of the design of turbo slits for the I20 beamline at Diamond.

2. Simple analytic design

Several derivations of the basic bending equations for flexural hinges have been studied and are compared with FEA results in a paper by Yong et al (2008). The various derivations are all refinements of the equations derived by Paros & Weisbord (1965).

The geometry of a simple 2D flexural hinge is defined here in Figure 1.
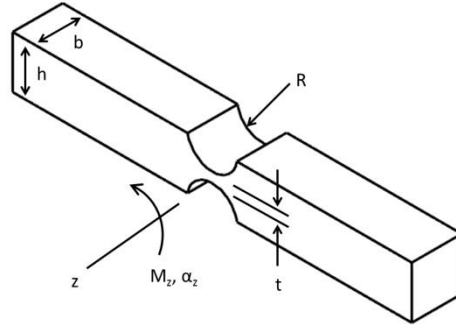

Fig.1 Definition of 2D flexural hinge

Paros & Weisbord derive exact equations from simple "beam theory", but of more use is their simplification for cases where hinge thickness t is much less than height h and cutting radius R.

$$\frac{\alpha_z}{M_z} = \frac{9\pi R^{1/2}}{2Ebt^{5/2}} \tag{1}$$

(Where α_z is the angular deflection, M_z is the moment applied, both around the z axis, and E which is the Young's Modulus of the material. All other variables are linear dimensions, defined in Figure 1)

For design purposes it is also important to know the maximum stress in the hinge. This may be easily calculated from simple beam theory as

$$\sigma_{max} = \frac{6M_z}{bt^2} \tag{2}$$

(Where σ_{max} is the maximum normal stress)
Or in terms of angular displacement

$$\sigma_{max} = \left(4/3\pi\right)\frac{\alpha_z Et^{1/2}}{R^{1/2}} \tag{3}$$

These equations may be easily programmed into a spreadsheet or other mathematical tools, or alternatively there is a freely available calculator, FlexHinge available on the internet for download (Web-2). Flexhinge uses empirical formulae (based on FEA analysis) for calculating stiffness and material stress values.

3. FEA validation

When we refer to "validating" the design using FEA, it should be noted that FEA itself is only a calculation tool, and is only as good as the model used, and the assumptions made. In particular for flexural hinges it is important that the mesh used provides an adequate number of mesh cells across the thickness of the hinge, while not wasting calculation time by having too dense a mesh in the main beam section. A suitable mesh is shown in Figure 2. For most cases the hinge can be analysed as a 2D structure (for b >> t) again optimizing resources.
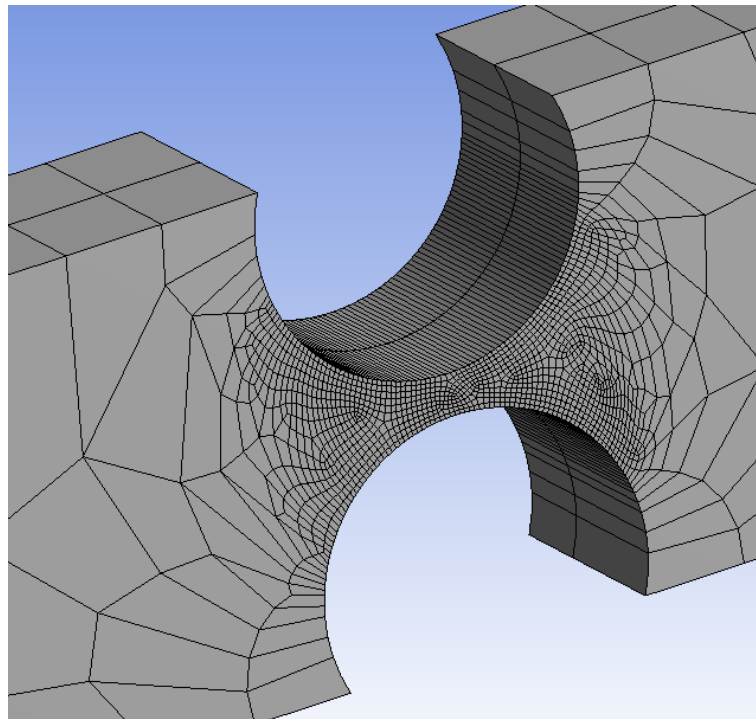

Fig.2 FEA mesh for flexural hinge

For the example case of an aluminium hinge with a 0.1N force applied in each case, the following values can be compared.

Table 1. Comparison of simple calculations and FEA analysis

Parameters					
D = hinge cut out diam (mm)	4.00	4.00	4.00	4.00	4.00
t = hinge thickness (mm)	0.10	0.20	0.30	0.40	0.50
2t/D (t/R)	0.05	0.10	0.15	0.20	0.25
FEA					
angle (rad)	0.1337	0.0246	0.0092	0.0046	0.0027
max stress (GPa)	0.8871	0.2452	0.1105	0.0630	0.0407
Calculation					
angle (rad)	0.1484	0.0262	0.0095	0.0046	0.0027
max stress (GPa)	1.0000	0.2500	0.1111	0.0625	0.0400
Difference (calculation relative to FEA)					
angle	11%	6%	4%	1%	-2%
max stress	11%	2%	1%	-1%	-2%

We note that the largest difference is for a small value of t/R. This results from a constant force being applied for each of the cases above, resulting in a large angular deflection for a small t/R. In more general terms, the greatest variation of simple theory from a fuller FEA analysis will occur either for a larger value of t/R or a larger angular deflection. This is reflected in the table illustrating a greater difference at the two extremes of values taken. This paper does not attempt to say in detail what conditions can be accurately predicted using simple analysis, though the results here suggest that reasonable solutions are obtained for angular distortion less than 10 mrad (0.57°)and t/R less than 0.25.

For precision mechanisms, for example for the design of synchrotron components, we often require to know the necessary force (for sizing motors) for a given displacement and the maximum stress in the hinge. In both these cases an accuracy of better than 5% is adequate. We also fundamentally require precision. It must be recognized that with flexure mechanisms there is a degree of parasitic movement. It may not be possible to calculate this precisely using a simple analytic method, only by using a finite element method.

4. Design considerations

Study of the equations 1-3 allows the basic dimensions of the hinge to be identified before modelling the hinge in a 3D CAD system and performing FEA. Many mechanisms used in precision instrumentation are displacement limited, i.e. we know how far the mechanism needs to move and it is necessary to size the joint and provide adequate force to obtain this. Often the mechanism is protected from overstressing the hinge by a physical limit stop on the displacement. In this case we observe that, in terms of the hinge thickness.

$$\sigma_{max} \propto t^{1/2}$$
$$Force \propto t^{5/2}$$

Alternatively the mechanism may be limited by the force available, in which case

$$\sigma_{max} \quad \propto \quad 1 / t^2$$
$$\alpha_z \quad \propto \quad 1 / t^{5/2}$$

In more general terms, Slocum (1992) identifies useful design considerations.

1. Stress ratio. As described above, the stress in the hinge may be the determining factor. Slocum suggests the maximum stress should be kept below about 15% of the yield strength of the material. Heat treatment may be considered to achieve the best yield strength. Although our analysis has concentrated on the 2D case, in a real mechanism there may be forces in the third dimension, which will lead to additional stresses, and which can be analysed using FEA.

2. Whether to use a monolithic or clamped design. We have assumed a monolithic design, but for heavier loads or larger deflections a clamped design may be more appropriate

3. Thermal. Due to the high surface area to volume ratio, flexural hinges are especially prone to thermal distortion, and so it is necessary to carefully consider the materials used, and the thermal environment.

4. Flexure kinematics. Initially the mechanism can be modelled as a simple pin jointed frame, and using an appropriate mechanism design software package for more complex geometries. However the assumption that the flexural hinge behaves as an idealized hinge is not exact, and for some geometries movement of the virtual centre may be critical. This will need to be modelled using FEA

5. Case Study

The I20 beamline at Diamond requires a set of vertical slits to be scanned across a horizontal fan over a travel range of 80mm. This requires the slits to be light weight and of small size due to the fast scanning requirements. The initial design solution identified was to use a 4 bar flexure mechanism such that a force applied in the y direction produced a motion in the x direction with an amplification factor of 1/10. This is illustrated in Figure 3. The geometrical arrangement requires an angular movement of 11 degrees for a slit movement of 0.5mm. Application of equations 1 & 2 for a hinge thickness of 0.2 mm shows a stress of 1921 MPa, significantly over the limit for Titanium 4, though the force and travel were within the capabilities of the chosen piezo actuator.
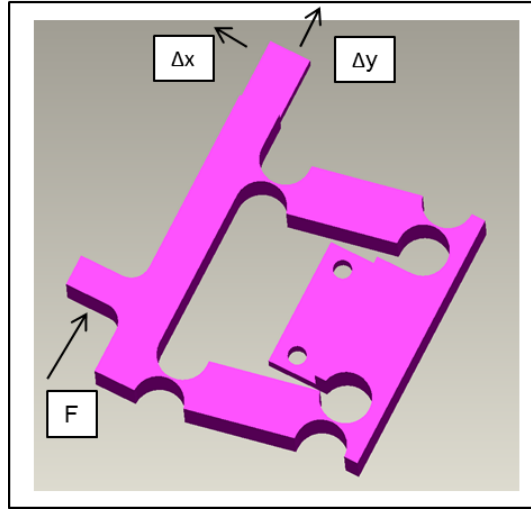
Fig.3 Preliminary design layout for slits mechanism

The design was reconsidered and it was decided that a better solution would be to have the direction of actuation in the same direction as the required slit movement. Lengths of the beams and the hinge parameters were varied to give a maximum stress of 77.1 MPa for an angular displacement of ± 0.36 degrees, to give the required 0.5mm total movement. The required force was 0.411 N. All these values are within the design constraints for motor performance and material stress limit. An FEA model was developed to validate these results. The following table 2 shows the level of agreement for this specific geometry.

Table 2. Results compared for final geometry.

	Equation	Single beam FEA	Full FEA model
Force (N)	0.411	0.477	0.472
Force (%)	-13%	1%	0%
Stress (MPa)	77.1	80.8	82.3
Stress (%)	-6%	-2%	0%

FEA on the full model calculated a stress of 82.3 MPa and a force of 0.472 N. The FEA results for a simplified single hinge is also presented. In all these cases t/R is 0.08, the angular deflection is ± 6 mrad and the material is Titanium 4. While there is a force difference of 13%, and a stress difference of 6%, the approach is clearly suitable for scoping the design before modelling the component as a solid model and carrying out a full FEA analysis.

Figure 4 shows the final design with the piezo actuator acting on the flexural hinge, and showing the slit blades and the limit switches.
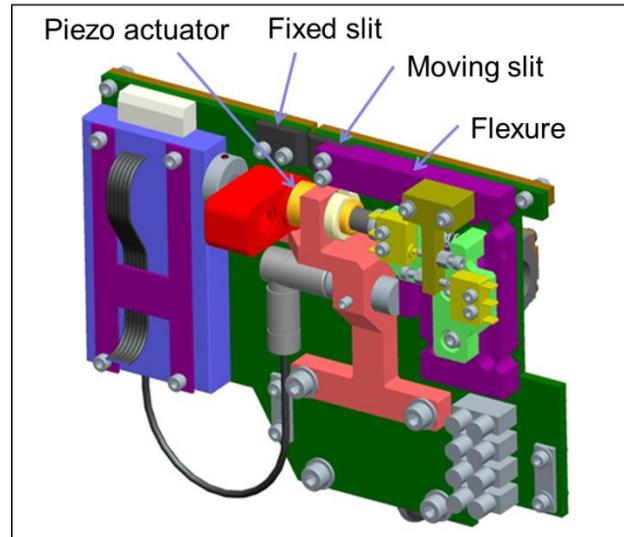
Fig.4 Final implementation for turbo slits

6. Conclusion

An analytic equation, derived from simple beam theory, can be used in scoping the design of a flexure mechanism, before fuller analysis using a FEA model. This approach has been demonstrated using the turbo slits from the I20 beamline at Diamond.

Acknowledgements

References

F S Eastman. The Design of Flexure Pivots, Journal of the Aeronautical Sciences (Institute of the Aeronautical Sciences), Vol. 4, No. 1 (1937) 16-21.

J Paros, L Weisbord, How to design flexure hinges. Machine Design 37 (1965) 151–156.

A H Slocum, Precision Machine Design, (1992) 521-538.

Y Yong, T-F Lu, D Handley, Review of circular flexure hinge design equations and derivation of empirical formulations. Precision Engineering 32 (2008) 63–70

Web sites:

Web-1: http://www.efunda.com/designstandards/plastic_design/hinge.cfm Consulted 03 November 2014.

Web-2: http://en.vinksda.nl/   Consulted 10 October 2014.